Truth Tables

We can model compound Boolean conditions using truth tables. Take, for example:

if (roll1 == 4 || roll2 == 4) {

//statements to be executed go here

}

The corresponding truth table is:

|  |  |  |
| --- | --- | --- |
| roll1 == 4 | roll2 == 4 | roll1 == 4 || roll2 == 4 |
| T | T | T |
| T | F | T |
| F | T | T |
| F | F | F |

In other words, if roll1 == 4 evaluates to T, roll2 == 4 evaluates to T, then we can look at the corresponding row of the truth table and see that

(roll1 == 4 || roll2 == 4) will also evaluate to T.

Often, instead of writing out entire statements like (roll1 == 4 || roll2 == 4) we will use variables (P,Q,R…), called predicates, when modeling this logic. Here is the general truth table for (inclusive) or:

|  |  |  |
| --- | --- | --- |
| **P** | **Q** | **P || Q** |
| T | T | T |
| T | F | T |
| F | T | T |
| F | F | F |

The other basic truth tables:

|  |  |
| --- | --- |
| **P** | **!P** |
| T |  |
| T |  |

|  |  |  |
| --- | --- | --- |
| **P** | **Q** | **P && Q** |
| T | T |  |
| T | F |  |
| F | T |  |
| F | F |  |

|  |  |  |
| --- | --- | --- |
| **P** | **Q** | **P xor Q** |
| T | T |  |
| T | F |  |
| F | T |  |
| F | F |  |

|  |  |  |
| --- | --- | --- |
| **P** | **Q** | **P == Q** |
| T | T |  |
| T | F |  |
| F | T |  |
| F | F |  |

To do more complicated truth tables, we generally create extra columns to help us through the intermediate steps.

**Example:** The truth table for !(!P && Q) is

|  |  |  |  |  |
| --- | --- | --- | --- | --- |
| **Q** | **P** | **!P** | **!P && Q** | **!(!P && Q)** |
| T | T | F |  |  |
| T | F | T |  |  |
| F | T | F |  |  |
| F | F | T |  |  |

**Example:** The truth table for P || (Q && R) is

|  |  |  |  |  |
| --- | --- | --- | --- | --- |
| **P** | **Q** | **R** | **Q && R** | **P || (Q && R)** |
| T | T | T |  |  |
| T | T | F |  |  |
| T | F | T |  |  |
| T | F | F |  |  |
| F | T | T |  |  |
| F | T | F |  |  |
| F | F | T |  |  |
| F | F | F |  |  |

**Example:** The truth table for (P && Q && R) || !P

|  |  |  |  |  |
| --- | --- | --- | --- | --- |
| **P** | **Q** | **R** | **P && Q && R** | **(P && Q && R) || !P** |
|  |  |  |  |  |
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|  |  |  |  |  |

**Number of rows:**

A truth table with one proposition P has 2 rows and a truth table with two propositions P and Q has 4 rows.

*How many rows are there in a truth table with 3 propositions?*

*4 propositions?*

*n propositions?*

DeMorgan’s Laws:

!(P && Q) =

!(P && R) =

Distributive Laws:

P && (Q || R) =

P || (Q && R) =

Tautology and Contradition:

Order of Operations:

1. ( )
2. !
3. == and !=
4. &&
5. ||

**Example:** Make a truth table for P && Q || P && R. Simplify the expression first using the distributive law.

|  |  |  |  |  |
| --- | --- | --- | --- | --- |
| **P** | **Q** | **R** |  |  |
|  |  |  |  |  |
|  |  |  |  |  |
|  |  |  |  |  |
|  |  |  |  |  |
|  |  |  |  |  |
|  |  |  |  |  |
|  |  |  |  |  |
|  |  |  |  |  |

**Example:** Prove one of DeMorgan’s Laws by making two truth tables and showing that both expressions are equivalent.

|  |  |  |
| --- | --- | --- |
| **P** | **Q** | **!(P && Q)** |
| T | T |  |
| T | F |  |
| F | T |  |
| F | F |  |

|  |  |  |
| --- | --- | --- |
| **P** | **Q** | **! P || ! Q** |
| T | T |  |
| T | F |  |
| F | T |  |
| F | F |  |